BACKYARD IMPROVEMENTS

- Pattern recognition
- Arithmetic sequences
- Square and triangular numbers
- Organizing and interpreting data

Getting Ready

What You'll Need

Pattern Blocks, 1 set per pair (only squares are needed)

Activity master, page 99

Overview

Students use Pattern Blocks to build increasingly larger squares and staircase models, and examine the underlying number sequence patterns. In this activity, students have the opportunity to:

- recognize patterns
- discover properties of arithmetic sequences
- learn about square and triangular numbers
- make predictions based on patterns
- compare sequences to see how they are related

Other *Super Source* activities that explore these and related concepts are:

Ripples, page 14

Marquetry, page 19

The Pyramid Mystery, page 24

Bees in the Trees, page 29

The Activity

On Their Own (Part 1)

Alan is planning to build a square patio in his backyard using square pieces of slate that measure 1 foot by 1 foot. He needs to decide what dimensions to make the patio so that he can determine how many pieces of slate to order. What information can you gather that might help Alan with his project?

- Use the orange Pattern Blocks to represent the square pieces of slate. Working with your partner, build models of increasingly larger square patios, beginning with the smallest possible patio, the one made from one square.
- Each time you build a new patio model, record the number of blocks you added to the previous model to build the next bigger square, the perimeter of the patio, and the total number of blocks in the new patio. Organize your data in a table.
- Record the data for the first ten squares, but build squares only until you discover a pattern that will produce all the numbers needed for your table.

- Look for relationships between the dimensions of the square patio, its perimeter, and the total number of pieces of slate needed. Think about how you might generalize your findings. For example, think about what the data would be for a patio with dimensions *n* feet by *n* feet.
- Be ready to discuss and justify your conclusions.

Thinking and Sharing

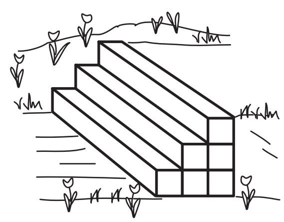
Have students help you create a class chart for the patios they modeled. Label the column headings: *side length of square patio, number of blocks added, perimeter,* and *total number of blocks*. If students have differing data, have them work together to come to agreement.

Use prompts like these to promote class discussion:

- What patterns do you see in the data?
- How would you describe the number sequence in the *number of blocks added* column?
- How would you describe the number sequence in the *perimeter* column?
- How would you describe the number sequence in the *total number of blocks* column?
- How many blocks would be needed for the 11th square? the 12th square? the *n*th square?
- How many blocks would be needed for the 20th square? How do you know?
- What algebraic expressions can you write to express the relationships you found?

On Their Own (Part 2)

What if... ***Alan also decides to build a set of stairs from one level of his backyard to another? He wants to use railroad ties for the steps, and must determine how many he needs. What information can you gather that might help Alan with this project?***



- Using the orange squares to represent the side view of the railroad ties, work with your partner to build models of increasingly larger staircases.
- Each time you build a new staircase, record the number of blocks you added to build the next taller step, the perimeter of the new side view, and the total number of ties needed to build the staircase. Organize your data in a table.
- Record the data for the first ten staircases, but build sets of stairs only until you discover a pattern that will produce all the numbers needed for your table.
- Look for relationships between the number of steps in the staircase, the perimeter of the side view, and the total number of railroad ties needed. Think about how you might generalize your findings to a staircase with *n* steps.
- Compare the sequences you found in the first activity with the sequences you found in this activity. Discuss your observations with your partner.

Thinking and Sharing

Have students help you create a chart similar to that in the first activity. Label the column headings: *number of steps in staircase, number of railroad ties added, perimeter,* and *total number of railroad ties*. Invite students to share their observations with the class.

Use prompts like these to promote class discussion:

- What patterns do you see in the data?
- How would you describe the number sequence generated by the entries in the *number of railroad ties added* column? the *perimeter* column? the *total number of railroad ties* column?
- How many railroad ties would be needed for the 11th staircase? the 12th staircase? the *n*th staircase?
- How many railroad ties would be needed for the 20th staircase? How do you know?
- What algebraic expressions can you write to express the the relationships you found?
- What did you notice when you compared the sequences from Part 1 with those from Part 2?

Describe how the various sequences you found in these activities were alike and how they were different. Make up some sequences of your own that are similar in some way to the ones that were generated by the data.

Teacher Talk

Where's the Mathematics?

The data students collect provide them with an opportunity to discover several types of sequences. The entries in the first column of each table form the sequence of consecutive positive integers, an example of an arithmetic sequence. Help students to see that each new term of an arithmetic sequence is generated by adding the same value to the previous term. In this case, that value (also called the *common difference*) is 1. The entries in the *perimeter* column of both tables also form an arithmetic sequence where the common difference between any two consecutive terms is 4. Students should notice that each term in this column is four times the number in the first column; therefore, the *n*th term in this column could be represented using the algebraic expression $4n$.

The table here shows the data pertaining to the materials needed to build square patios.

side length of square patio	number of blocks added	perimeter	total number of blocks
1	1	4	1
2	3	8	4
3	5	12	9
4	7	16	16
5	9	20	25
6	11	24	36
7	13	28	49
8	15	32	64
9	17	36	81
10	19	40	100
⋮	⋮	⋮	⋮
n	$2n - 1$	$4n$	$n \times n$, or n^2

The entries in the second column (1, 3, 5, 7, 9, ...) also form an arithmetic sequence. The common difference between any two consecutive terms is 2. Students may recognize this sequence as the sequence of positive odd integers. Each number in this sequence is 1 less than twice the side length of the corresponding square. Thus, to find the value of the *n*th term in this sequence, the algebraic expression $2n - 1$ can be used.

The sequence 1, 4, 9, 16, 25, 36, ... , generated by the data in the last column, is the sequence of perfect squares. Students should recognize that since the length and width of a square are always equal, and the total number of pieces of slate needed to build a square patio is the product of the length and width, that total will necessarily be a square number. Thus, to find the number of pieces of slate needed to build a patio with side length of n feet, calculate n^2.

It is important to point out that the sequence of perfect squares is not an arithmetic sequence; the difference between terms is not constant. However, the sequence formed by these differences *is* arithmetic, an interesting and intriguing characteristic of the sequence of perfect squares. Students will have opportunities to learn more about sequences that behave this way when they study them later on in high school algebra.

As students look for relationships among the sequences in the table and consider the relationship between the values in the 2nd and 4th columns, they may discover that square numbers can always be generated by adding consecutive odd numbers beginning with 1. For example, adding the first four odd integers (1 + 3 + 5 + 7) gives 16, or 4^2; adding the first seven odd integers (1 + 3 + 5 + 7 + 9 + 11 + 13) gives 49, or 7^2. This interesting fact may prompt students to explore other patterns related to perfect squares.

The table below shows the data pertaining to the materials needed to build staircases.

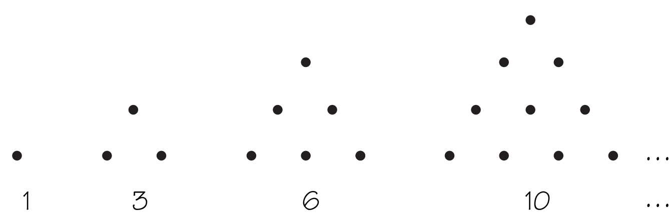
number of steps in staircase	number of railroad ties added	perimeter	total number of railroad ties
1	1	4	1
2	2	8	3
3	3	12	6
4	4	16	10
5	5	20	15
6	6	24	21
7	7	28	28
8	8	32	36
9	9	36	45
10	10	40	55
⋮	⋮	⋮	⋮
n	n	$4n$	$\frac{n(n+1)}{2}$

Students should discover that with each new step, the number of railroad ties added is 1 larger than the previous number added. They may also notice that, for example, if it takes 5 more railroad ties to build the 5-step staircase than to build the 4-step staircase, then it will take 6 more ties to build the 6-step staircase from the 5-step staircase. Algebraically speaking, to build an *n*-step staircase, *n* additional railroad ties will need to be added to the previous structure.

To find the total number of railroad ties needed to build a set of stairs, students may resort to counting the ties each time they add an entry to the table. Some students may realize that they are simply adding the consecutive integers from 1 up to and including the number of ties they've added to form the new staircase. For example, to build a 7-step staircase, the total number of railroad ties needed will be 1 + 2 + 3 + 4 + 5 + 6 + 7, or 28. Finding a way to represent such a sum algebraically may prove challenging for some students. Some students may suggest the expression $n + (n - 1) + (n - 2) + (n - 3) + ... + 1$. Others, through trial-and-error techniques, may be able to discover the relationship between n and the sum of the first n counting numbers and derive the expression for the sum: $\frac{n \text{ x } (n + 1)}{2}$. Some students may need to be given this formula and asked to confirm it using their their data.

The sequence generated by the entries in the *total number of railroad ties* column (1, 3, 6, 10, 15, 21, ...) is known as the sequence of triangular numbers. Each term is the sum of consecutive integers. Beyond that, it is interesting to note that by adding together any two consecutive terms of the triangular sequence, a term from the sequence of perfect squares is generated. The triangular numbers can also be generated by using counters (or other manipulatives) to build triangular arrays as shown below; hence the name "triangular" numbers.

triangular numbers as
equilateral triangle arrays



triangular numbers as
right triangle arrays