EVEN - STEVEN

- Sampling
- Making and testing hypotheses
- Fairness

Getting Ready

What You'll Need

Snap Cubes, 10 each of three different colors per pair

Paper bags, 3 per pair

Activity Master, page 98

Overview

Students create random samplings with Snap Cubes to test games for fairness. Students then create their own game and verify its fairness using sampling techniques. In this activity, students have the opportunity to:

- draw conclusions based on a sample
- collect and analyze data
- make decisions about fairness

Other *Super Source* activities that explore these and related concepts are:

Snapshot, page 9

Color Draw, page 13

The Activity

On Their Own (Part 1)

Steven has designed three versions of a two-player game called Even-Steven. He wants each player to have the same chance of winning so that his game is fair. Which, if any, of Steven's game variations are fair?

- Players take turns drawing two Snap Cubes from a paper bag. Player 1 scores one point if the colors are the same; Player 2 scores one point if the colors are different. The player with the most points after 20 draws is the winner.
- Predict which of the following versions are fair:

  Version 1: The bag holds 1 Snap Cube of one color and 2 Snap Cubes of another color.

  Version 2: The bag holds 2 Snap Cubes of one color and 2 Snap Cubes of another color.

  Version 3: The bag holds 1 Snap Cube of one color and 3 Snap Cubes of another color.
- Decide who is Player 1 and who is Player 2. Now play Version 1. Here's how:
  - ◆ Put the Snap Cubes in the bag for Version 1.
  - ◆ Take turns drawing two Snap Cubes from the bag.
  - ◆ Record the score, and then put the cubes back in the bag.
  - ◆ Continue until you have completed 20 trials.

- *Now change the contents of the bag. Repeat the activity for Version 2 and Version 3.*
- *Use your data to decide which variations of the game are fair. Be ready to explain your reasoning.*

Thinking and Sharing

Have pairs report their results for Version 1 and indicate whether they believe the version is fair. Create a chart like the one shown below.

Same (Player 1)	Version 1 Different (Player 2)	Conclusion
7	13	Unfair
9	11	Fair
5	15	Unfair

Do the same for Versions 2 and 3.

Use prompts like these to promote class discussion:

- Did the sampling results match your predictions?
- How did you decide whether the version was a fair one? Explain any additional mathematical procedures you may have used.
- Does the compiled class data support the decision your pair made about the fairness of the version? Why or why not?
- Using the compiled class data, which versions, if any, are fair games?
- Are there any other methods, besides sampling, that can be used to evaluate the versions? Explain.

On Their Own (Part 2)

What if... ***you wanted to design a variation of*** **Even-Steven** ***that uses three colors of Snap Cubes? Use the same rules for scoring as you did in the first three versions. How many cubes of each color would you put in the bag to make a fair game?***

- *Working with your partner, decide how many Snap Cubes of three colors to put in the bag. Use up to 10 cubes of each color.*
- *Test the game by making 20 draws and recording your scores.*
- *If you believe the game is fair, repeat the game at least two more times to collect more data.*

- If the game seems unfair, make adjustments to the contents of the bag and try again.
- Be ready to justify why you believe your version is a fair one.

Thinking and Sharing

Have pairs share their versions and display their data.

Use prompts like these to promote class discussion:

- How did you decide how many of each color to put in the bag at first? Did you use any mathematical procedures or strategies to help you decide?
- If you concluded that your first version was not a fair game, how did you decide what changes to make?
- How do you know that your final version is a fair one? Explain.
- What would you expect to happen if a fair version were played ten times?

For Their Portfolio

Steven decides to try another version using 4 red and 2 blue Snap Cubes. Without actually playing the game, describe what you expect the results would be. Do you think the version would be a fair one? Write a summary explaining your reasoning.

Teacher Talk

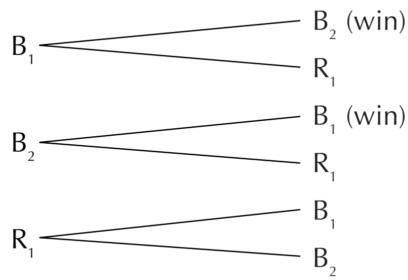
Where's the Mathematics?

This sampling activity gives students the opportunity to see the relationship between theoretical data and experimental (empirical) data. Students may be surprised that the third version makes *Even-Steven* a fair game. Intuition might indicate that the second version is the fair one since it contains two of each color and most people associate equality with fairness.

Theoretically, in order for a game such as *Even-Steven* to be fair, both players have to have an equally likely chance of winning. That means that the number of possible win outcomes must equal the possible number of no-win outcomes.

Tree diagrams, lists, and matrices provide ways to find out whether one outcome is as likely to happen as another. To analyze the outcomes, cubes of the same color must be distinguished from each other. In this tree diagram for Version 1, B_1 and B_2 represent two blue cubes and R_1 represents one red cube. The tree diagram indicates the six possible outcomes, two that win and four that do not. Version 1 is unfair since the probability of a match is 2/6 or 1/3 and the probability of a non-match is 4/6 or 2/3.

Version 1 Outcomes



Students may prefer to make an organized list of possible outcomes. Again, numbers are used to distinguish between cubes of the same color. Since each cube in Version 2 can be paired with three other cubes, there are 12 pairs, or permutations. It doesn't matter which cube of the pair is counted first; therefore, half of these are duplicates and can be crossed off the list, leaving six different pairs, or combinations. These combinations are:

Version 2 Outcomes			
B_1, B_2 (win)	~~B_2, B_1 (win)~~	~~R_1, B_1~~	~~R_2, B_1~~
B_1, R_1	B_2, R_1	~~R_1, B_2~~	~~R_2, B_2~~
B_1, R_2	B_2, R_2	R_1, R_2 (win)	~~R_2, R_1 (win)~~

The list shows that there are two ways to make a match. The probability of a match is 2/6 or 1/3. Likewise, the probability of a non-match is 4/6 or 2/3. Therefore Version 2 is also unfair because the likelihood of a non-match is more probable than that of a match.

Another way to analyze the possible outcomes is with a matrix—a grid in which each cube in the bag is listed once along the top and once down the side. Here is a matrix showing the outcomes for Version 3. Again, duplicates are crossed off.

Version 3 Outcomes				
	B1	R1	R2	R3
B1	X	no win	no win	no win
R1	~~no win~~	X	win	win
R2	~~no win~~	~~win~~	X	win
R3	~~no win~~	~~win~~	~~win~~	X

The Xs in the matrix represent the four impossible outcomes. Same color pairs are marked *win,* indicating that the probability of a match is 3 out of 6, or 1/2 . Since the same probability exists for a non-win, Version 3 is the only fair version of the game.

When students try using three colors in Part 2, the only fair version of *Even-Steven* uses the colors in a 9:3:1 ratio. Students who use any of the methods described above may find the correct ratio. Students may use theoretical probability to calculate the probability of drawing a matching pair. First, calculate the theoretical probability of drawing a pair of each of the three colors. To do this, imagine drawing the cubes one at a time. Multiply the chance of drawing a color first by the chance of drawing the same color a second time on the next draw. Then, add to find the probability of drawing a pair of any one of the three colors.

Example: 9 red, 3 blue, and 1 yellow

Chance of drawing red on first draw: 9/13
Chance of drawing another red on second draw: 8/12 or 2/3
(8, because 1 of the 9 reds has been removed. 12, because 1 of the 13 cubes has been removed.)
Multiply to find the combined probability of drawing a red pair: 9/13 x 2/3 = 18/39 or 6/13

Chance of drawing blue on first draw: 3/13
Chance of drawing another blue on second draw: 2/12 or 1/6
Multiply to find the combined probability of drawing a blue pair: 3/13 x 1/6 = 3/78 or 1/26

Chance of drawing yellow on first draw: 1/13
Chance of drawing another yellow on second draw: 0
Multiply to find the combined probability of drawing a yellow pair: 1/13 x 0 = 0

Add: 6/13 + 1/26 + 0 = 36/78 + 3/78 = 39/78 = 1/2

There is a 1 in 2 chance of drawing a pair. Likewise, there is an equally likely chance of not drawing a pair. As students explore making changes to make a fair game, they may be surprised at how “uneven” the colors must be to result in a nearly fair game.