FROM HEAD TO TOE

- Measurements
- Estimation
- Ratios and proportions
- Decimals

Getting Ready

What You'll Need

Snap Cubes, about 100 per pair

Cuisenaire Rods, 2 of each color per pair

Calculators (optional)

Activity Master, page 110

Overview

Students measure various lengths of the human body using Snap Cubes and Cuisenaire Rods. They also set up ratios and compare their measurements to the Golden Ratio. In this activity, students have the opportunity to:

- estimate measurements to the nearest whole
- set up ratios and proportions
- convert fractions to decimal equivalents
- learn about the Golden Ratio

Other *Super Source* activities that explore these and related concepts are:

Fill 'Em Up!, page 64

Rain Gear, page 72

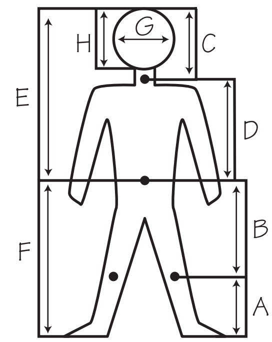
The Activity

On Their Own (Part 1)

Mathematicians have found that many of the ratios formed using the lengths of specific body parts are equivalent to something called the "Golden Ratio." Using Snap Cubes, how can you find out if your measurements are made up of Golden Ratios?

- Work with a partner. Decide who will be measured first.
- Use Snap Cube strands, formed by joining cubes end-to-end, to measure specific lengths on the body. If a particular measurement is not made up of a whole number of cubes, approximate the length to the nearest whole number.
- Using a Snap Cube strand, find the number of cubes needed to represent Distance A, from your partner's knees to his or her toes (floor). Record the number of cubes required.

- Continue the process of finding and recording the number of cubes needed for:
  - Distance B, from the waist to the knees
  - Distance C, from the top of the head to the mid-neck
  - Distance D, from the mid-neck to the waist
  - Distance E, from the top of the head to the waist
  - Distance F, from the waist to the toes (floor)
  - Distance G, across the face, from cheekbone to cheekbone
  - Distance H, from the top of the head to the bottom of the chin



- Exchange roles with your partner and repeat the activity.
- The ratio 1.618:1 is a close approximation of the Golden Ratio. Each person should place his or her measurements into the following fractions, convert the fractions to decimals and then compare them to the Golden Ratio.

$$\frac{\text{Distance B}}{\text{Distance A}} \quad \frac{\text{Distance D}}{\text{Distance C}} \quad \frac{\text{Distance F}}{\text{Distance E}} \quad \frac{\text{Distance H}}{\text{Distance G}}$$

- Compare your results with those of your partner. Be ready to explain your findings and discuss whether your measurements match that of the Golden Ratio.

Thinking and Sharing

Students may find the measurements taken to their toes are more accurate if shoes or sneakers are removed. Remind students that they should measure the vertical distances, not the distances along the contours of the body. Invite pairs to discuss their findings.

Use prompts like these to promote class discussion:

- What was hard about measuring the distances? What was easy? Why?
- How did you compare the four ratios to the Golden Ratio?
- How accurate were your findings?
- Why might your ratios vary from the Golden Ratio?
- If you were told a person's height, how could you use the Golden Ratio to determine specific distances on the body without measuring?

On Their Own (Part 2)

What if... you compare lengths along the human face? Can you discover any measurements that form Golden Ratios?

- Work with your partner. Decide who will be measured first.
- Using Cuisenaire Rods, find the rod that best approximates each length on your partner's face. Remember each color of Cuisenaire Rod is a different length and their lengths increase by 1 centimeter at a time. If any length is longer than the orange rod (10 cm), use a combination of rods to find the desired measure. For each measurement below, record the color of the rod(s) used and its length:
  - Distance J, from the top of the head to the bridge of the nose
  - Distance K, from the bridge of the nose to the bottom of the nose
  - Distance L, from the bottom of the nose to the mouth opening
  - Distance M, from the mouth opening to the bottom of the chin
  - Distance N, from the bottom of the nose to the bottom of the chin
  - Distance P, from the bridge of the nose to the mouth opening
  - Distance Q, from the bridge of the nose to the bottom of the chin
- Exchange roles with your partner and repeat the entire set of measurements.
- Using the measurements for your own face, find which distances could be paired together so that their ratio approximates the Golden Ratio. Record each pair of measures and be ready to justify your conclusions.

Thinking and Sharing

Ask students to share the ratios they found that best matched the Golden Ratio. Make a list on the chalkboard of the facial distances used and the decimal approximations of their ratios.

Use prompts like these to promote class discussion:

- What was hard about measuring the distances? What was easy? Why?
- How did you go about choosing which distances to use in the ratios?
- What ratios did you choose not to calculate? Why?
- What ratios best approximated the Golden Ratio?
- How did your findings compare to other students' results?

The Golden Ratio (or Divine Proportion) has captured people's interest for thousands of years. Research its origin and applications and write a paragraph describing what you discovered.

Teacher Talk

Where's the Mathematics?

Teachers may choose to have students complete *For Their Portfolio* prior to working on the activities so they have a better understanding of the history and significance of the Golden Ratio (also known as the Divine Proportion). Students may enjoy sharing their research with classmates and gaining a more comprehensive appreciation of the ratio's unique qualities and its role in ancient cultures as well as in today's world.

As students begin to take each other's measurements in the first activity, they may discover that they can get better readings by having their partner hold one end of the Snap Cube strand in place while they add or subtract cubes from the other end of the strand. It is important to remind students to use vertical measurements in computing their ratios.

Because the heights of the students in the class will be different, the number of cubes needed for the distances found in the first activity will vary. Upon calculating the decimal equivalents of the four ratios, though, some students may find that their results are remarkably close to the Golden Ratio, approximately 1.618. To find the decimal equivalent of each body distance ratio, divide each numerator by its corresponding denominator.

If the two quantities are equal, or even accurate to one or two decimal places, then it can be concluded that the ratio of these body distances is an example of a Golden Ratio.

However, some ratios may not be as close as others to the value of the Golden Ratio. Inaccurate measuring or approximating the number of cubes needed may affect the decimal value. Or students may hypothesize that the Golden Ratio, which is based on adult body distances, may not be valid for students who have not reached their potential adult height. Even adult measurements will not always reflect the Golden Ratio; the correlations are based on "average" body types.

The second activity requires students to experiment with various combinations of numbers. When the Golden Ratio is expressed (approximately) as 1.618/1, the numerator is greater than its denominator. Because of this, students can discard those ratios in which the denominator is equal to or greater than the numerator, and focus attention on ratios that are greater than 1. They may also notice that whether Snap Cubes or Cuisenaire Rods are used as the unit of measure for the body lengths, the ratios will be similar.

The ratios using facial lengths that may best approximate the Golden Ratio are,

from the first activity: $\frac{\text{Distance H}}{\text{Distance G}}$

from the second activity: $\frac{\text{Distance N}}{\text{Distance K}}$ and $\frac{\text{Distance K}}{\text{Distance L}}$