PYTHAGORAS DELIVERS THE MAIL

- Properties of triangles
- Pythagorean Theorem
- Area

Getting Ready

What You'll Need

Geoboards, 1 per student

Rubber bands

Rulers

Dot paper, page 119

Activity master, page 105

Overview

Students examine squares built on the sides of right, obtuse, and acute Geoboard triangles. They look for relationships between the areas of the squares and the type of triangle on which they are built. In this activity, students have the opportunity to:

- devise methods for finding areas
- formulate and test generalizations
- learn about and apply the Pythagorean Theorem
- use mathematical reasoning to solve a real-world problem

Other *Super Source* activities that explore these and related concepts are:

Napkins and Place Mats, page 35

Polygons, Pegs, and Patterns, page 44

The Airline Connection, page 48

Inside Out, Outside In, page 52

The Activity

On Their Own (Part 1)

Pythagoras, one of the world's most famous mathematicians, discovered interesting relationships between types of triangles and the lengths of their sides. Can you retrace the steps leading to his remarkable discovery?

- Work with a group of at least 4 students. Each of you should make a different-sized right triangle on your Geoboard.
- Record your triangle on dot paper near the center of the paper.
- Using a ruler, draw a square on each side of your triangle. Make the sides of each square congruent to the side of the triangle on which it is built.
- Find and record the area of each of your three squares. Let the area of one small dot-paper square be the unit of measure.
- Discuss and check the work of the other members of your group.
- Repeat the activity using obtuse triangles and then acute triangles.
- Look for relationships among the areas of the three squares surrounding each type of triangle. Generalize the findings of your group.

Thinking and Sharing

Invite groups to share their recordings and their observations about the areas of the squares they built on the sides of the different types of triangles. Encourage them to help each other find the areas of squares they may have had difficulty working with.

Use prompts like these to promote class discussion:

- What method(s) did you use to find the areas of your squares?
- For which squares was it hard to find the area? For which was it easy? Why?
- What patterns did you discover?
- What generalizations can you make about the areas of the three squares built on the sides of a right triangle? an obtuse triangle? an acute triangle?
- Do you think your findings would be consistent for other triangles of the same type? How do you know?

You may want to tell students that the relationship that holds for right triangles (the sum of the areas of the squares on the legs is equal to the area of the square on the hypotenuse) is called the *Pythagorean Theorem,* discovered by and named for Pythagoras.

On Their Own (Part 2)

What if... ***a letter measuring 1/8" by 11" by 14" is sent to a teacher who has an open-front rectangular mailbox in the school office measuring 5" high by 10" wide by 15" deep. Will the letter fit into the teacher's mailbox without being bent or folded?***

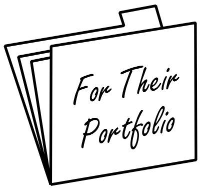
- Draw a model of the front of the mailbox on dot paper.
- Discuss with your group how the letter might be positioned in the mailbox, and sketch it on your dot paper drawing.
- Using what you learned in the first activity, decide whether the mailbox can accommodate the teacher's letter. Be ready to justify your conclusions.

Thinking and Sharing

Invite groups to share their conclusions and tell how they reached them.

Use prompts like these to promote class discussion:

- What information was needed to draw the mailbox model on dot paper?
- How did you choose to position the letter in the mailbox? Why?
- How did you apply what you learned in the first activity to solve this problem?
- What did you conclude about whether the letter would fit? Explain your reasoning.
- What size restrictions would you place on any piece of mail if it is to fit in the mailbox?



Write a paragraph or two describing how you might approach the mailbox problem if the mail was a package instead of a letter.

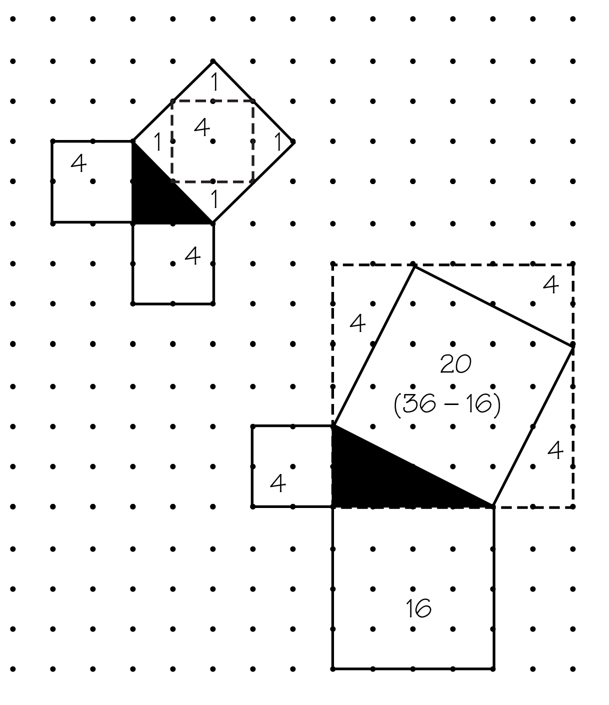
Teacher Talk

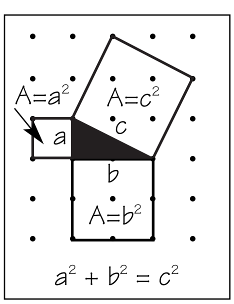
Where's the Mathematics?

Students should be familiar with the concepts of *right triangles* (those containing a 90° angle), *acute triangles* (those in which each angle measures less that 90°), and *obtuse triangles* (those containing an angle whose measure is greater than 90°). Working in groups of at least four, students should be able to build and investigate a good number of each of these types of triangles.

As students build squares on the sides of their triangles, they will discover that some of the squares are more easily constructed than others. They also may use different strategies to find the areas of their squares. For those squares that have sides parallel to the edges of the dot paper, some students may simply count the number of unit squares contained in the squares. Other students may recognize that the areas of these squares can be found by multiplying the length of one side of the square by itself.

To find the area of a square whose sides are not parallel to the edges of the dot paper, some students may partition the square into shapes whose areas may be easier to find. Others may enclose them inside larger squares whose sides *are* parallel to the edges of the dot paper. The areas of these "enclosed squares" can then be determined by finding the area of the larger, surrounding square and subtracting the areas of the four corner triangular areas. Examples of these methods are shown at the right.

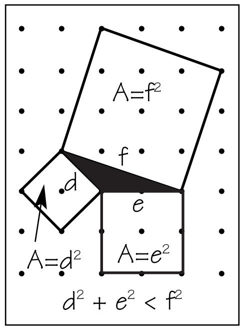




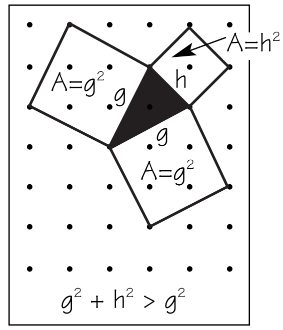
right triangle

In examining the areas of the squares surrounding each right triangle, students should notice that the sum of the areas of the squares on the two shorter sides (the legs) is equal to the area of the square on the longest side (the hypotenuse). This relationship is known as the *Pythagorean Theorem* and can be summarized as follows: In a right triangle, if a and b are the lengths of the legs and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$.

For obtuse triangles, students will find that the area of the square on the longest side of the triangle is greater than the sum of the areas of the two smaller squares. For acute triangles, students should find that the area of the square on the longest side of the triangle is less than the sum of the areas of the two smaller squares. If the acute triangle is isosceles or equilateral, there may not be a "longest side," but the same inequality will exist based on the relative lengths of the sides.

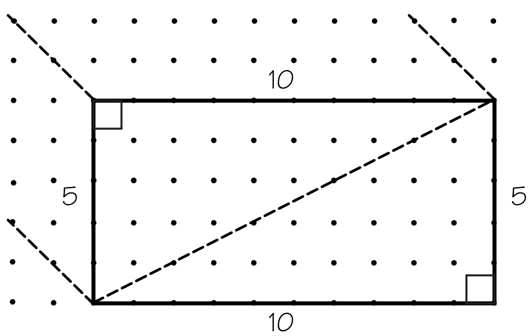


obtuse triangle



acute isosceles triangle

Part 2 of the activity gives students opportunities to work on a practical application of the Pythagorean Theorem and to build a mathematical model that can be used as an aid in analyzing the problem. Students may realize that the length of the diagonal of their dot paper rectangle is the width of the widest piece of mail that will fit into the mailbox. It is also the hypotenuse of two congruent right triangles.



By applying the Pythagorean Theorem to either one of the right triangles, students can find the area of the square that can be built on the diagonal ($5^2 + 10^2 = 125$ square units), and then consider the length of the diagonal itself, keeping in mind that the diagonal is the side of this square. They may estimate that the side length is between 11 and 12 units. Students who use a calculator may take the square root of 125 to arrive at the decimal approximation 11.18. Either way, students can conclude that the 11" width of the letter can be accommodated in the mailbox.

The depth of the mailbox might be considered extraneous information in this problem because students might reason that a piece of mail could extend past the opening of the mailbox. Also, if the piece of mail was significantly thicker than 1/8", the letter might need to be less than 11" wide to fit in the mailbox, as the thickness would prevent it from fitting into the corners.