RIPPLES

- Pattern recognition
- Growth patterns
- Arithmetic sequences
- Writing algebraic expressions

Getting Ready

What You'll Need

Pattern Blocks, 1 set per pair

Colored pencils or markers

Activity master, page 100

Overview

Students investigate the pattern of growth that occurs when one Pattern Block is surrounded by blocks of the same kind, and each successive design is then surrounded. In this activity, students have the opportunity to:

- discover patterns
- make predictions
- investigate arithmetic sequences
- use algebraic expressions to generalize patterns

Other *Super Source* activities that explore these and related concepts are:

Backyard Improvements, page 9

Marquetry, page 19

The Pyramid Mystery, page 24

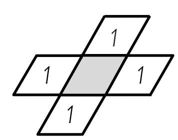
Bees in the Trees, page 29

The Activity

On Their Own (Part 1)

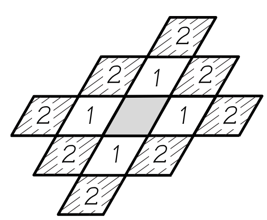
Mariano notices that when a stone is thrown into a calm body of water, it produces a ripple effect of larger and larger circles whose centers are the stone's point of contact with the water's surface. Mariano wants to investigate a similar kind of ripple effect using Pattern Block shapes. What patterns can be discovered in a sequence of ripples?

- Work with a partner. Using the Pattern Block square or the blue or tan rhombus as the "stone," completely surround your stone with blocks of the same shape to form the first "ripple." Record your design. Color it using one color for the original stone and another color for the blocks in the first ripple.



Design 1

- Surround your first ripple design with more blocks of the same shape to form the second ripple. Make sure that the entire perimeter of the ripple is surrounded with new blocks. Record your new design. Color the new ripple with a different color.



Design 2

- Predict how many stones it will take to form the third ripple and check your prediction.
- As each new ripple is generated, record the number of blocks in the new ripple, the total number of blocks in the ripple design, and the perimeter of the ripple design.
- Continue predicting, surrounding your designs, and recording the data until you have built the sixth ripple.
- Look for patterns in your results. See if you can write algebraic expressions that generalize your findings.
- Repeat the activity using yellow Pattern Block hexagons.

Thinking and Sharing

Ask students who used the square for their stone to work together to prepare a class chart showing their data. Do the same for pairs who used the blue rhombus and those who used the tan rhombus. Also ask some pairs to work on a class chart showing the data for the hexagon. Have all four charts available for the class discussion.

Use prompts like these to promote class discussion:

- What do you notice about the data in the class charts?
- What did you notice about the numbers of blocks needed to surround the different-shaped stones and their ripples?
- How would you describe the patterns generated by the entries in the different columns of the charts?
- What relationship do you see between the number of sides on the original stone shape and the growth pattern of its ripples?
- How did you go about making your predictions?
- What predictions would you make for the 10th ripple for each shape?
- What algebraic expressions did you write to express the relationships you found?

On Their Own (Part 2)

What if... ***Mariano decides to investigate the ripple patterns for the triangle and trapezoid stones? Will he find that these shapes produce patterns similar to the ones he found for the other shapes?***

- Using the procedure from the first part of the activity, gather data about ripples formed from using the Pattern Block triangle as the stone.
- Compare the patterns formed by the triangle data with those you found in Part 1.
- Predict what will happen if you use the Pattern Block trapezoid for the stone. Test your prediction by building the first three ripple designs. As you add trapezoids to the design, be sure to join short edge to short edge and long edge to long edge.
- Discuss whether or not the trapezoid designs follow growth patterns that are similar to those of the other shapes. Be ready to justify your findings.

Thinking and Sharing

Ask students to add the charts for the triangle and trapezoid shapes to those already displayed. Students may have difficulty in completing the chart for the trapezoid, but ask them to fill in as much as possible.

Use prompts like these to promote class discussion:

- What patterns did you find in your data?
- What similarities/differences exist between the data from the first activity and that from the second? How can you explain this?
- What happened when you tried to build ripple designs around the trapezoid stone? Why do you think this happened?
- What predictions would you make for the 10th ripple in the triangle sequence? the 10th ripple in the trapezoid sequence?

For Their Portfolio

Write a letter to Mariano explaining how the shape of a stone affects the patterns formed by its ripples. Use examples and drawings to help illustrate your explanation.

Teacher Talk

Where's the Mathematics?

Students enjoy having the opportunity to formulate and test predictions based on their findings. The patterns in the sequences of numbers generated by the ripples emerge almost immediately and enable students to predict, check, and verify their findings for the square (or rhombus) and hexagon. The charts for these shapes are shown below.

square or rhombus

ripples	number of blocks added	total number of blocks	perimeter
1	4	5	12
2	8	13	20
3	12	25	28
4	16	41	36
5	20	61	44
6	24	85	52
⋮	⋮	⋮	⋮
n	$4n$	$4\left[\frac{n(n+1)}{2}\right]+1$	$8n+4$

hexagon

ripples	number of blocks added	total number of blocks	perimeter
1	6	7	18
2	12	19	30
3	18	37	42
4	24	61	54
5	30	91	66
6	36	127	78
⋮	⋮	⋮	⋮
n	$6n$	$6\left[\frac{n(n+1)}{2}\right]+1$	$12n+6$

Students should recognize that the number of sides on each shape determines how many blocks will be required to surround it the first time. The second ripple requires twice as many; the third ripple, three times as many, and so on.

As students experiment with algebraic expressions that will generate specific terms in a particular sequence, they may recognize that many of the patterns form arithmetic sequences. For the square (or rhombus), the entries in the *number of blocks added* column forms an arithmetic sequence in which the common difference is 4, while the entries in the same column for the hexagon form an arithmetic sequence whose common difference is 6. Students may also notice that these entries are multiples of the number of the ripple under consideration. They may see that to find the number of blocks added for a ripple around the square or rhombus, they can multiply the ripple number by 4. To find the number of blocks added for a ripple around a hexagon, they can multiply the ripple number by 6.

The entries in the *perimeter* column for the square (or rhombus) and the hexagon also form arithmetic sequences whose common differences are 8 and 12, respectively. Encourage students to search for relationships between the ripple number and the perimeter. If, for example, they identify the common difference in the sequence of perimeters for the square to be 8, they may, through test-and-check techniques, be able to determine that the perimeter in each case is 4 more than 8 times the number of ripples. Thus, the perimeter of a design with n ripples will be $8n + 4$. Similarly, the perimeter of designs built around the hexagon can be expressed as $12n + 6$, where n is, again, the number of ripples. Note that both of these expressions are equivalent to the general expression $s(2n + 1)$, where s is the number of sides of the original polygon.

Finding an algebraic expression that generates the entries in the *total number of blocks* columns may prove challenging for many students. Some students may notice that the terms in these sequences are each 1 more than a multiple of 4 (in the case of the quadrilaterals) and 1 more than a multiple of 6 (in the case of the hexagon), the 1 accounting for the original stone. This may prompt them to investigate the multipliers of 4 and 6 being used. Interestingly, these multipliers are the triangular numbers: 1, 3, 6, 10, ... (See *Backyard Improvements*, page 9, for more on triangular numbers.) Each triangular number is the sum of consecutive counting numbers. Students may be familiar with the formula for the sum of the first n counting numbers, $\frac{n \times (n + 1)}{2}$, and may be able to incorporate it into an expression representing the total number of blocks needed to build a design with n ripples surrounding, for example, a hexagon: $6 \times \left[\frac{n \times (n + 1)}{2}\right] + 1$. Some students may need help deriving this expression, but can be asked to use their data to verify it.

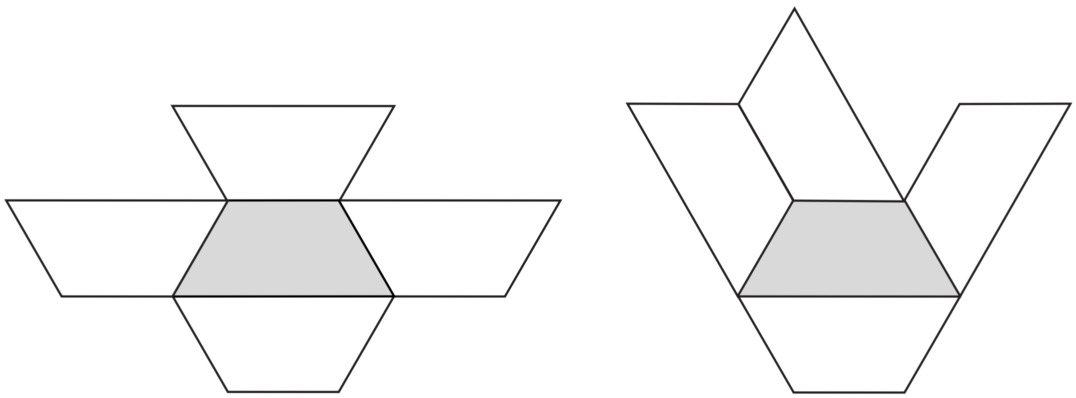
Some different occurrences emerge in Part 2 of the activity. The chart for the triangle is shown below.

triangle

ripples	number of blocks added	total number of blocks	perimeter
1	3	4	6
2	6	10	12
3	9	19	15
4	12	31	21
5	15	46	24
6	18	64	30
⋮	⋮	⋮	⋮
n	$3n$	$3\left[\frac{n(n+1)}{2}\right] + 1$	—

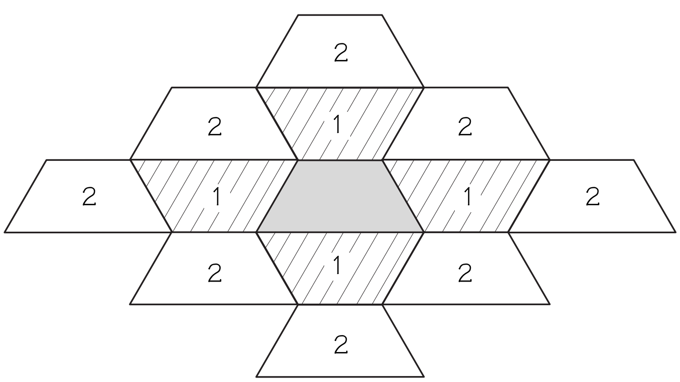
Many of the same patterns and types of sequences found in Part 1 exist in this chart, with the exception of the entries in the *perimeter* column. Students may notice that if the ripple number is even, 6 was added to the previous entry, while if the ripple number is odd, the value added was 3. Students may be able to discover why these values increase in an alternating pattern. For example, when the single triangle is surrounded, each of the surrounding blocks covers only one side of the original block, leaving 2 exposed sides on the new outer perimeter. It will take 6 triangles to surround these exposed sides for the second ripple. However, to surround the second ripple, there are three places where a single triangle will cover exposed sides of two different triangles. Therefore, 3 fewer triangles will be needed. This construction pattern continues for every pair of consecutive ripples.

When students attempt to build ripples around the trapezoid, they may experience confusion about knowing how to place the blocks. Even as early as in the first ripple, students will discover that there are several ways of placing the surrounding blocks.

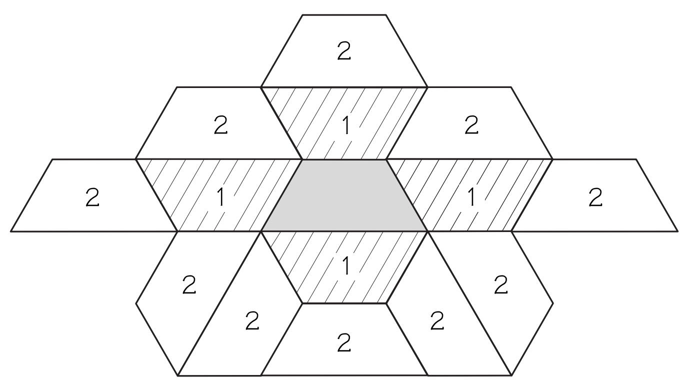


Two possible arrangements for first ripple

There are even more options for arrangements for the second ripple, and students may discover that not all second ripples require the same number of blocks. Given this situation, no consistent number patterns will emerge in the trapezoid data.



8 blocks needed for second ripple



10 blocks needed for second ripple

